QUASIHYPERBOLIC METRIC AND QUASISYMMETRIC MAPPINGS IN METRIC SPACES

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Abstract. In this paper, we prove that the quasihyperbolic metrics are quasi-invariant under a quasisymmetric mapping between two suitable metric spaces. Meanwhile, we also show that quasi-invariance of the quasihyperbolic metrics implies that the corresponding map is quasiconformal. At the end of this paper, as an application of above theorems, we prove that the composition of two quasisymmetric mappings in metric spaces is a quasiconformal mapping.

1. Introduction

where

(1.2) L_f

In 1990, Väisälä studied quasiconformal mappings between infinite-dimensional

Using the same assumptions as in Theorem 1.9, by combing Theorem 1.9 and

Definition 2.3. Let γ be a rectifiable curve in an open set $G \subset X$. The *quasihyperbolic length* of γ in G is

$$l_{qh}(\gamma) = \int_\gamma \frac{ds}{\delta_G(x)}.$$

The *quasihyperbolic distance* between x and y in G is defined by

Proof.

(1) By Observation 2.6, we know that G is recti¯ably connected. For any recti¯-

Therefore, it follows

$$l_{qh}(\circ$$

(2) Let $a \in$

Let $X;Y$ be $c;c^0$-quasiconvex metric spaces and let G $X;G$

Hence, by the de¯nition of quasisymmetry, we have

$$jf(y) \; ¡ \; f(c)$$

From the Step 1 of the proof of Theorem 1.6, we know that f has $\big(2H^2(H+1); 3\big)$-ring property. In view of the fact $3r_j < \pm(x$

where $\tilde{\mu}$

Step 5.1. We show that

$$\mathrm{dist}\left(f(B_0), G'\setminus f(\alpha_0 U_0)\right) > 3R \quad \text{and} \quad 3R < \delta_{G'}\big(f(x)\big). \tag{5.2}$$

From Fact 3.1, it follows that

$$G'\setminus f\big(\alpha_0 U_0\big) \neq \emptyset.$$

Suppose that $y_0 \in f(B_G)$ and $z \in G'\setminus f\big(\alpha_0 U$

6. Appendix

For the sake of completeness, we give an example to show that the assumption of non-cut-point in Theorem 1.8 is necessary.

Example 6.1. For each positive integer $n \geq 1$, we define the functions $f_n(x)$ on $[0, 1]$ as follows:

$$f_n(x) = \begin{cases} nx & \text{for } x \in [0, \frac{1}{2}] \\ x + \frac{n-1}{2} & \text{for } x \in [\frac{1}{2}, 1]. \end{cases}$$

Let $X = \mathbb{R} = Y$ and $G = (0, \infty) = G'$. Define a homeomorphism f :

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References

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